Hot-Air Balloon

At the West Texas Balloon Festival, a hot-air balloon is sighted at an altitude of 800 feet and appears to be descending at a steady rate of 20 feet per minute. Spectators are wondering how the altitude of the balloon is changing as time passes.

1. What function relating the variables best describes this situation?

2. Make a table of values and/or graph to show the balloon's altitude every 5 minutes beginning at 5 minutes before the balloon was sighted until the balloon lands.

3. How high was the balloon 5 minutes before it was sighted?

4. How long does it take the balloon to reach an altitude of 20 feet? How long does it take the balloon to land?

5. A second balloon is first sighted at an altitude of 1200 feet and is descending at 20 feet per minute. How does the descent and landing time of the second balloon compare with that of the first balloon? What does this mean graphically?

6. A third balloon is first sighted at an altitude of 800 feet but is descending at 30 feet per minute. How does the descent and landing time of the third balloon compare with that of the first balloon? What does this mean graphically?

7. At the instant the first balloon is sighted, a fourth balloon is launched from the ground rising at a rate of 30 feet per minute. When will the first and fourth balloon be at the same altitude? What is that altitude? What does this mean graphically?

Teacher Notes

Materials:
One graphing calculator per student

Connections to Algebra I TEKS and Performance Descriptions:

(b.1) Foundations for functions.
The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.

The student:
(A) describes independent and dependent quantities in functional relationships;

(C) describes functional relationships for given problem situations and writes equations or inequalities to answer questions arising from the situations;

(D) represents relationships among quantities using concrete models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities; and

(E) interprets and makes inferences from functional relationships.

(c.1) Linear functions.
The student understands that linear functions can be represented in different ways and translates among their various representations.

The student:
(A) determines whether or not given situations can be represented by linear functions.

Scaffolding Questions:

- What are the constants in the problem? What quantities vary?
- What quantity will be the dependent variable? The independent variable?
- What kind of function models the situation? How do you know?
- What decisions must you make to build a table for the function?
- What decision must you make to graph the function?
- How can you find the balloon's height at any given time?
- How can you find the time it takes the balloon to reach a given height?

Sample Solution:

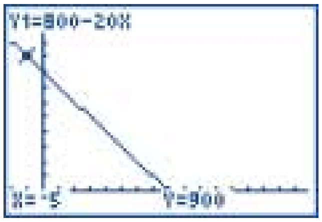
1. The starting height, 800 feet, is decreased by 20 feet per minute. The height, h, equals 800 minus 20 times the number of minutes, m.

   $h = 800 - 20m$

2. The time 5 minutes before it was sighted is represented by -5.

m	$800 - 20m$	h
-5	800 – 20(-5)	900
0	800 – 20(0)	800
5	800 – 20(5)	700
10	800 – 20(10)	600
15	800 – 20(15)	500
20	800 – 20(20)	400

The graph may also be used to examine the situation.



3. The value of y is 900 when x is -5. Therefore, the balloon was at 900 feet 5 minutes before it was first sighted.

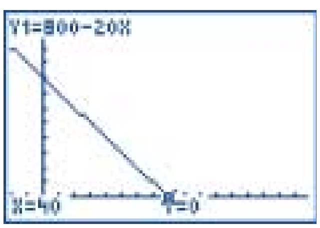
4. Solve for m:

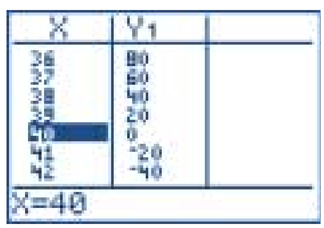
$$\begin{aligned} 800 - 20m &= 20 \\ -20m &= -780 \\ m &= 39 \end{aligned}$$

It takes the balloon 39 minutes to descend to 20 feet above the ground.

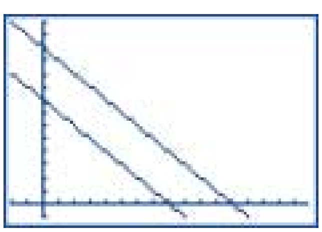
Solve $800 - 20m = 0$ for m to get $m = 40$. The balloon lands in 40 minutes.

The graph or table may also be examined to determine when the height is 0.





5. The balloon is at a higher altitude but descending at the same rate. It will take longer to land. The second function is $y = 1200 - 20x$. The graphs have different y-intercepts and x-intercepts. The graphs will be parallel lines since they have the same slope.



(c.2) Linear functions.
The student understands the meaning of the slope and intercepts of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations.

The student:
(B) interprets the meaning of slope and intercepts in situations using data, symbolic representations, or graphs;

(C) investigates, describes, and predicts the effects of changes in m and b on the graph of y = mx + b;

(D) graphs and writes equations of lines given characteristics such as two points, a point and a slope, or a slope and y-intercept;

(E) determines the intercepts of linear functions from graphs, tables, and algebraic representations;

(F) interprets and predicts the effects of changing slope and y-intercept in applied situations.

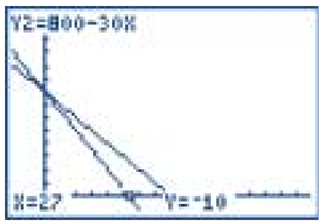
(c.3) Linear functions.
The student formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

The student:
(A) analyzes situations involving linear functions and formulates linear equations or inequalities to solve problems;

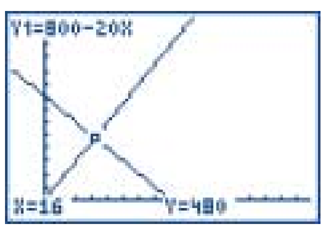
(B) investigates methods for solving linear equations and inequalities using concrete models, graphs, and the properties of equality, selects a method, and solves the equations and inequalities.

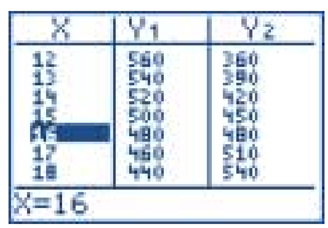
6. The third balloon starts at the same height as the first but is descending faster. Therefore, the third balloon will land sooner. The third function rule is $y = 800 - 30x$.

   The graphs have the same y-intercept but different x-intercepts. The x-intercept for the third balloon is less than that of the first balloon. The graph for the third balloon's descent will be steeper than that for the first balloon.



7. The function for the fourth balloon is $y = 30x$. To see if they are ever at the same altitude, explore with tables or graphs, or solve $800 - 20x = 30x$ to get $x = 16$. Sixteen minutes into descent/launch, both balloons will be at the same height, 480 feet.





Extension Questions:

- If the function of the motion of a fifth balloon had been $y = 700 - 20x$, how would the movement of the balloon have been different from the first?

The balloon would have been sighted at a height of 700 feet instead of 800 feet. The rate of descent would have been the same as the rate of descent of the first balloon.

Texas Assessment of Knowledge and Skills:

Objective 1:
The student will describe functional relationships in a variety of ways.

Objective 3:
The student will demonstrate an understanding of linear functions.

Connections to Algebra I: 2000 and Beyond Institute:

II. Linear Functions
1 Linear Functions
  1.2 The Y-Intercept
  1.3 Exploring Rates of Change

Connections to Algebra End-of-Course Exam:

Objective 2:
The student will graph problems involving real-world and mathematical situations.

Objective 3:
The student will write linear functions (equations of lines) to model problems involving real-world and mathematical situations.

- Would the fifth balloon have landed sooner or later than the first balloon? Explain how you know.

If it started at a lower altitude and descended at the same rate, it would land sooner. The x-intercept would be 700 divided by 20, or 35 seconds.

Student Work

1) y = 800 - 20x
starting height · 20 feet per minute

2)

feet y	minutes x	
900	-5 ←	5 minutes before sited at 900
800	0 ←	when it was sited
700	5 ←	5 minutes after it was sited
600	10 ←	10 minutes after sited
500	15 ←	15 minutes after sited
400	20 ←	20 minutes after sited

3) 900 feet in the air from the table
I made a graph on the calculator and traced

4) 39 minutes at x = 20
40 minutes at x = 0

5) The second balloon is going later because its starting at 1200 and the first one was sited at 800 There is a (400 feet difference)

6) The Third balloon is descending at 30 ~~feet~~ per minute and the first one was going 20 feet per minute so the Third balloon is going to get to the ground faster

7) The 4th and 1ST balloon are going to meet at 16 ~~feet~~ minutes at 480 feet. I graphed y = 800 - 20x and y = 30x and found the intersection